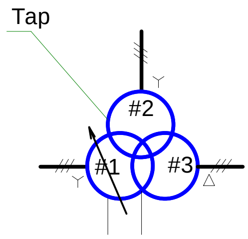
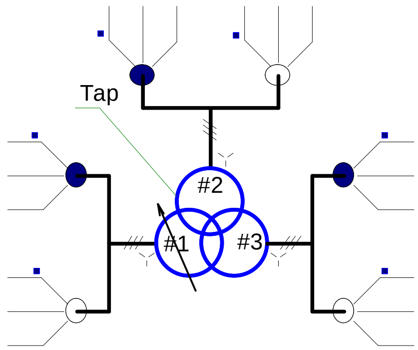
3-Phase 3-Winding Transformer

Description

Input parameters





Description

This component models a 3-phase, 3-winding core-type transformer and is based on the magnetic equivalent electrical circuit approach [1], [3]

Options are provided so that the user may model the core as a three-limb or a five-limb transformer (inter-phase coupling is represented in this model). The saturation characteristic can be directly entered as a I-V curve, where the current and voltage values correspond to the RMS quantities measured during test. These values are internally converted to peak values by means of an algorithm that estimates and corrects for the harmonic content in the magnetizing current due to saturation of the core material [2]. If desired, the saturation of the core can be eliminated altogether. In this case the transformer will still model the magnetic coupling between phases. As in the UMEC model, some elements of the core geometry (i.e. core type, yoke and winding limb dimension ratios, etc.) are required as data input as well.

Other options include choosing between modeling the core saturation by means of the current injection method or through an embedded method. In the magnetic equivalent electrical circuit approach, each of the sections of core are represented as non-linear inductances. The non-linearity of such inductances can be modelled using the parallel current injection method in order to account for the saturation. In this method, a constant inductance is placed in parallel with a current source that emulates the higher requirements of current as the inductance saturates. Alternatively, in the embedded method, the inductances are changed every time there is a change in saturation level, therefore affecting the EMTDC [Y] matrix. The later method can be slower at times when compared with the current injection method. It is also expected to be more numerically stable.

The option for pre-established connections (Y or Delta) are offered. For non-standard connections, the option of ‘open ends’ can be selected. This will allow the user to connect the ends of the windings as required.

More: The magnetic equivalent circuit approach

Input Parameters

Configuration

General			
Transformer name	Text		Just an identifier. A name should be entered here to avoid compilation warnings
3-Phase Transformer MVA	Real	Constant	The 3-phase apparent power of the transformer [MVA]
Base operation frequency	Real	Constant	The base frequency of the electrical system in which the transformer resides [Hz]
No load losses	Real	Literal	Enter the total no-load losses (also known as core losses) for the transformer [pu]. Note: Equivalent resistances are calculated and placed in parallel with each element of the core (yokes and legs) in proportion to the volume of each section.
Copper losses	Real	Literal	Enter the total copper losses (also known as load losses) for the transformer in [pu]. Note: Equivalent winding resistances are calculated based on this entry so that losses are evenly distributed among all the windings. These resistances are placed in series with each of the windings. If a different distribution of the losses among windings is required, it is recommended to enter this value as zero, and manually place the winding resistances in series with each of the windings.
Model saturation?	Choice		Select **Yes** or **No** to enable or disable saturation
Transformer core construction	Choice		Select 3-Limb or 5-Limb core configuration (note: 3-phase bank option is not offered with this model)
Tap settings			
Tap changer on winding	Choice		Select the winding number for the on-line tap changer (or none).
Tap changing at any current	Choice		This option allows to select if the tap changing occurs at the time step when the Tap signal is changed, or only at the zero crossing of the current flowing through each phase tap winding
Transformer connections			
Winding # type	Choice		Select the connection type for the specified winding # - Star (Y), Delta (D) or “open ends”. The last option provides access to both ends of the wining allowing the user to connect the windings in whichever configuration is desired.
Delta # leads or lags Y	Choice		Select whether the D-connected winding voltage will lead or lag the Y-connected winding voltage by 30°

Winding Voltages

General			
Winding # Line to Line voltage (RMS)	Real	Constant	The Line to Line voltage rating of the respective winding [kV]
Winding #: Voltage across the winding (RMS)	Real	Constant	For open-ends connection only. The voltage rating across each individual winding [kV]

Transformer Impedances and Tap Settings

General			
Tap winding lower voltage [pu]	Real	Constant	Defines the lower per-unit voltage accepted in the tap winding. It must be at least 0.01 pu. This value is used in the calculation (interpolation) of the impedance swing for intermediate tap settings. See note below.
Tap winding upper voltage [pu]	Real	Constant	Defines the upper per-unit voltage accepted in the tap winding. It must be at least 0.01 pu. This value is used in the calculation (interpolation) of the impedance swing for intermediate tap settings Note: Most tap windings would model a fixed winding in series with a tap winding as a single winding. In these cases, the tap voltage range will vary, for example, between 0.9 and 1.1 pu for a 20% total variation over the nominal voltage. If an individual tap winding (with 100% variation) is modeled and it is required to model its negative swing, then the recommended method is to select the open ends connection, and to externally model the changeover switch.
Leakage reactance (#1 - #2)			
Positive seq. Impedance (#-#) Lower Tap	Real	Constant	The positive sequence leakage reactance of the transformer between the specified windings at the lower tap setting. This can be calculated based on short-circuit test results [pu]
Positive seq. Impedance (#-#) V nominal	Real	Constant	The positive sequence leakage reactance of the transformer between the specified windings at the nominal tap setting. This can be calculated based on short-circuit test results [pu]
Positive seq. Impedance (#-#) Upper Tap	Real	Constant	The positive sequence leakage reactance of the transformer between the specified windings at the upper tap setting. This can be calculated based on short-circuit test results [pu]

Saturation Curve

General			
Magnetizing current at rated voltage	Real	Constant	The percentage of primary winding current that flows through the transformer magnetizing susceptance. This value can be calculated based on the open-circuit test results [%]
Model type	Choice		Select 'Embedded' or 'Current Injection'. This allow to select different methods for simulating the saturation of the core. Through current injections parallel to the core saturable inductances, or by changing the inductance values (embedded method) directly in the EMTDC [Y] matrix
Current			
Point # - Current as percentage	Real	Constant	Enter the I-coordinates of the I-V curve. These points can be non-zero if desired. The maximum number of points is 10 [%]
Voltage			
Point # – Voltage in p.u.	Real	Constant	Enter the V-coordinates of the I-V curve. These points can be non-zero if desired. The maximum number of points is 10 [pu]

Core Aspect Ratios

See Figure 8 and Figure 9

General			
Ratio yoke/winding-limb length	Real	Constant	Enter the ratio of core yoke length to the core winding-limb length.
Ratio yoke/winding-limb area	Real	Constant	Enter the ratio of core yoke area to the core winding-limb area.
Ratio yoke/outer-limb length	Real	Constant	Enter the ratio of core yoke length to the core outer -limb length (yokes plus limb height).
Ratio yoke/outer-limb area	Real	Constant	Enter the ratio of core yoke area to the core outer-limb area.

Output Variables (for I-V input)

1. Core Currents [kA]			
Icore Phase A,B…	Real	Output	Name for the equivalent core circuit current in kA
2. Core fluxes [kWb]			
Winding A flux	Real	Output	Name for the corresponding section of core flux in kWb

Output Variables (for B-H input)

1. H [kA-t/m]			
H Phase A,B,C,	Real	Output	Name for the magnetic field intensity for the corresponding section of core
2. Core fluxes [T]			
Winding A flux density	Real	Output	Name for the corresponding section of core flux density in Tesla

The magnetic equivalent electrical circuit approach

The core equivalent electrical circuit

The magnetic equivalent electrical circuit approach adds an additional winding to the transformer (n+1) which will serve as the connection to the core electrical equivalent circuit.

A three-legged three-phase core-type transformer has a core construction as shown in Figure 1. For this type of construction, the areas of limbs and yokes are usually the same throughout the construction of the transformer core.

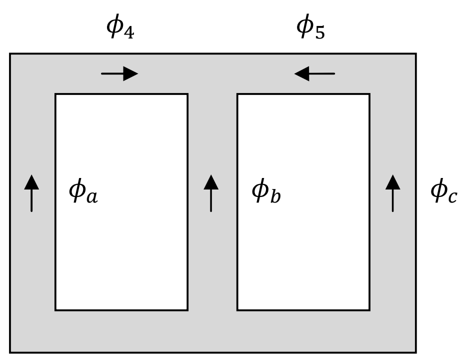


Figure 1 Core fluxes in a three-limb core-type transformer

The magnetic equivalent circuit for the transformer shown above is represented in the following figure

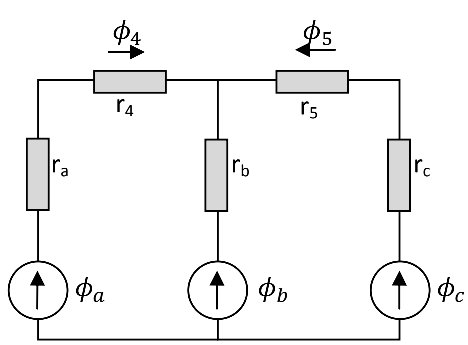


Figure 2 Lumped magnetic elements in three-limb core-type transformer

Similarly to the voltage loop equation in an electrical circuit, the sum all of the magneto-motive forces along any closed loop of the equivalent magnetic circuit is equal to zero.

$$\sum \mathcal{F} = 0 \quad \text{Eq. 1}$$

Therefore, the magneto-motive force $\mathcal{F}$ loop equations for the circuit above are

$$\begin{aligned} \mathcal{F}_a{}' - \mathcal{F}_b{}' &= \phi_a r_a + \phi_4 r_4 - \phi_b r_b \\ \mathcal{F}_b{}' - \mathcal{F}_c{}' &= \phi_b r_b - \phi_5 r_5 - \phi_c r_c \end{aligned} \quad \text{Eq. 2}$$

The summation of fluxes (node equations) gives the following equation

$$\phi_a + \phi_b + \phi_c = 0 \quad \text{Eq. 3}$$

Also

$$\begin{aligned} \phi_a &= \phi_4 \\ \phi_c &= \phi_5 \end{aligned} \quad \text{Eq. 4}$$

Eq. 2 can be re-written in terms of magneto-motive forces only as

$$\begin{aligned} \mathcal{F}_a{}' - \mathcal{F}_b{}' &= \mathcal{F}_a + \mathcal{F}_4 - \mathcal{F}_b \\ \mathcal{F}_b{}' - \mathcal{F}_c{}' &= \mathcal{F}_b - \mathcal{F}_5 - \mathcal{F}_c \end{aligned} \quad \text{Eq. 5}$$

Or in terms of the currents, assuming that all the magneto-motive forces are produced by coils with equal number of turns ($\mathcal{F} = Ni$),

$$\begin{aligned} i_a{}' - i_b{}' &= i_a + i_4 - i_b \\ i_b{}' - i_c{}' &= i_b - i_5 - i_c \end{aligned} \quad \text{Eq. 6}$$

Similarly, Eq. 3 and Eq. 4, can be written in terms of voltages following the assumption that all the fluxes are linked by coils having the same number of turns N, $(v = N\, d\phi/dt)$

$$\begin{aligned} e_a + e_b + e_c &= 0 \\ e_a &= e_4 \\ e_c &= e_5 \end{aligned} \quad \text{Eq. 7}$$

Eq. 6 and Eq. 7 can be written or built in terms of an equivalent electrical circuit. Such equivalent circuit is shown in the figure below.

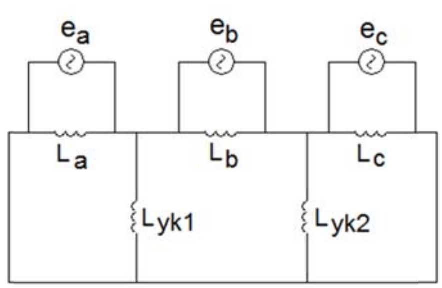


Figure 3 Equivalent electrical circuit of the magnetic circuit of a three-limb three-phase power transformer

It can be proven that the impedances in the equivalent electric circuit follow the inductance equation where the voltage is proportional to the derivative of the current ($v \propto di/dt$), therefore they can be represented as inductances. The non-linear behaviour of the core is integrated in the model by making such inductances non-linear.

The same procedure can be carried out for the five-limb core type transformer. Its equivalent electrical circuit is given in Figure 4

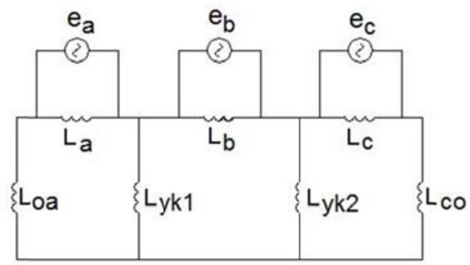


Figure 4 Equivalent electrical circuit of the magnetic circuit of a five-limb three-phase power transformer

Calculation of the Core Equivalent Circuit Inductances

The method outlined in this section explains how the inductances in the core equivalent circuit are calculated.

Calculating the inductances in the core equivalent circuit is a straightforward procedure when the actual core dimensions and number of turns in the windings are known. The core inductances would be given by the equation

$$L = \frac{n^2}{\mathcal{R}} \quad \text{Eq. 8}$$

Where is the n is the number of turns, one in this case, and $\mathcal{R}$ is the reluctance of the magnetic material. Therefore, the linear inductance of the core would be given by

$$L = \frac{\mu_R \times \mu_0 \times Area}{length} \quad \text{Eq. 9}$$

However, in most of the cases, obtaining this type of information can prove quite difficult. Transformer manufactures usually do not disclose internal construction details, unless requested at the time of purchase. Therefore, it can be even more difficult when modeling old transformers already connected to the network. In most cases, all the information available is limited to what is specified on the nameplate and the test reports.

The following sections explain the procedure used inside the PSCAD component, in order to model the transformer using just the available information in the two sources above mentioned, name plate and test reports. Additional information regarding the core aspect ratios is also needed.

Connection of the core equivalent circuit to the transformer model

The core equivalent electrical circuit is connected to the network through ideal single-phase transformers (one per phase) that incorporate the short-circuit behaviour of the power transformer into the model (see Figure 5) plus the voltage ratios between windings [6].

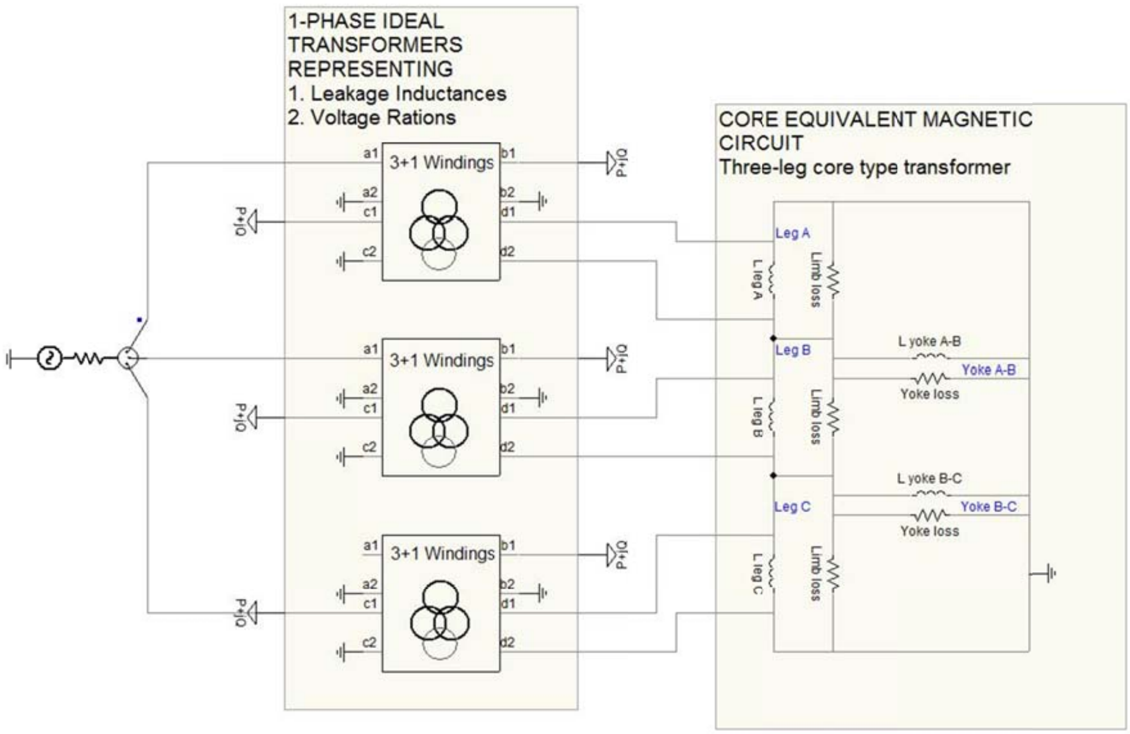


Figure 5 Connection of the core equivalent magnetic circuit to the electrical network through single-phase N+1 ideal transformers

Construction of the Single-Phase ideal transformers

The single-phase ideal transformers in Figure 5 are represented by inverse inductance matrices $[L]^{-1}$ of size (nxn), where n is the total number of windings including the added winding to the core (4 = 3+1 for Figure 5), such that they follow the equation

$$[v]_{n\times 1} = [L]_{n\times n}\frac{d}{dt}[i]_{n\times 1} \qquad \text{Eq. 10}$$

The steps for building the inverse inductance matrix are given in [7] and are the following

1. Build a reduced per-unit impedance matrix $[Zr^{pu}]$ of size (n-1 x n-1) such that

$$\begin{bmatrix} V_1 - V_n \\ V_2 - V_n \\ \vdots \\ V_{n-1} - V_n \end{bmatrix} = \begin{bmatrix} Zr^{pu}{}_{11} & Zr^{pu}{}_{12} & \cdots & Zr^{pu}{}_{1,n} \\ Zr^{pu}{}_{21} & Zr^{pu}{}_{22} & \cdots & Zr^{pu}{}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ Zr^{pu}{}_{n-1,1} & Zr^{pu}{}_{n-2,1} & \cdots & Zr^{pu}{}_{n-1,n-1} \end{bmatrix} \begin{bmatrix} I_1 \\ I_1 \\ \vdots \\ I_{n-1} \end{bmatrix} \qquad \text{Eq. 11}$$

where

V_i-V_n is the drop between winding i and winding n, and

$$Zr^{pu}{}_{j,k} = \frac{1}{2}\left(IX_{j,n} + IX_{k,n} - IX_{j,k}\right) \quad \text{with} \quad j=1,..,n-1 \text{ and } k=1,..,n-1 \qquad \text{Eq. 12}$$

$IX_{j,k}$ is the per-unit impedance voltage (or per-unit impedance) between windings j and k

For a four-winding transformer with per-unit winding impedances

$$IX_{12} = 0.05 \qquad \text{Eq. 13}$$
$$IX_{13} = 0.12$$
$$IX_{14} = 0.04$$
$$IX_{23} = 0.122$$
$$IX_{24} = 0.042$$
$$IX_{34} = 0.002$$

The reduced per-unit inductance matrix would be given by

$$Zr^{pu} = \begin{bmatrix} 0.122 & 0.057 & 0.002 \\ 0.057 & 0.042 & 0.002 \\ 0.002 & 0.002 & 0.002 \end{bmatrix} \qquad \text{Eq. 14}$$

2. Find the reduced inverse per-unit admittance matrix [Yr^{pu}] as

$$Yr^{pu} = [Zr^{pu}]^{-1} \qquad \text{Eq. 15}$$

For the matrix given in Eq. 14, the reduced per-unit admittance matrix is

$$Yr^{pu} = \begin{bmatrix} 22.535 & -30.986 & 8.451 \\ -30.986 & 67.606 & -36.62 \\ 8.451 & -36.62 & 528.169 \end{bmatrix} \qquad \text{Eq. 16}$$

3. The full size nxn per-unit atmitance is calculated using the equations

$$Y^{pu}{}_{j,k} = Yr^{pu}{}_{j,k} \quad for \;\; j,k \le n-1 \qquad \text{Eq. 17}$$
$$Y^{pu}{}_{k,n} = Yr^{pu}{}_{n,k} = -\sum_{j=1}^{n-1} Yr^{pu}{}_{k,j} \quad for \;\; k \ne n$$
$$Y^{pu}{}_{n,n} = -\sum_{j=1}^{n-1} Y^{pu}{}_{j,n}$$

For the reduced per-unit admittance matrix given in Eq. 16 the corresponding nxn admittance matrix is

$$Y^{pu} = \begin{bmatrix} 22.535 & -30.986 & 8.451 & 7.105\text{x}10^{-15} \\ -30.986 & 67.606 & -36.62 & 4.974\text{x}10^{-14} \\ 8.451 & -36.62 & 528.169 & -500 \\ -7.105\text{x}10^{-15} & 2.842\text{x}10^{-14} & -500 & 500 \end{bmatrix} \qquad \text{Eq. 18}$$

4. The Y^{pu} matrix can then be finally be converted to inverse the inductance matrix $[L]^{-1}$ format, as required by the EMTDC solver, by applying the following conversion

$$[L]^{-1} = Y^{pu}{}_{j,k}\,\omega \frac{Sbase}{V_j V_k} \qquad \text{Eq. 19}$$

If the winding voltages in the example above are

$$[V] = \begin{bmatrix} 132.79 \\ 66.39 \\ 7.96 \\ 7.96 \end{bmatrix} kV \qquad \text{Eq. 20}$$

The corresponding inverse inductance is then

$$[L]^{-1} = \begin{bmatrix} 10.06 & -44.168 & 100.467 & 8.447\text{x}10^{-14} \\ -44.168 & 192.747 & -870.781 & 1.183\text{x}10^{-12} \\ 100.467 & -870.781 & 1.048\text{x}10^{-5} & -9.916\text{x}10^{4} \\ -8.447\text{x}10^{-14} & 6.758\text{x}10^{-13} & -9.916\text{x}10^{4} & 9.916\text{x}10^{4} \end{bmatrix} \frac{1}{H} \quad \text{Eq. 21}$$

Core equivalent circuit lumped inductances

The calculation of the core equivalent circuit lumped inductances is based on the UMEC approach.

The calculation of the core inductances is a straightforward procedure when having clear data about the internal construction of the transformer, such as core dimensions, number of turns in the windings, etc. The core inductances would be given as

$$L_{CORE} = n^2 \frac{\mu_R \mu_0 Ac}{l} \quad \text{Eq. 22}$$

Where

n : is the number of turns in the winding. It would be assumed that the core winding is surrounded by one turn winding, therefore

$$L_{CORE} = \frac{\mu_R \mu_0 Ac}{l} \quad \text{Eq. 23}$$

μ_R : is the lumped relative permeability of the section of core

μ_0 : is the permeability of air

Ac : is the area of the section of core under consideration, and

l : is the length of the lumped section of core (core leg or core yoke)

In most cases, this type of information, such as core dimensions, number of turns on the actual transformer windings, and so on is not available to the final user and can prove very difficult if not impossible to obtain. Therefore, the transformer component provided in PSCAD was designed such that the available information in the nameplate of the transformer, plus some in the test report to be sufficient. The only additional information needed is the core aspect ratios, which only refer to approximate dimensions (not actual dimensions) of the core, just as is done in the UMEC transformer models.

The conversion procedure from magnetization test data Voltage vs. Current into core equivalent inductances is outlined below. This method is based on the Unified Magnetic Equivalent Circuit approach, a complete explanation on this approach can be found in [4], [5].

Since the multiple windings are dealt by the single-phase ideal transformer connected to the core equivalent circuit, then it is only necessary for the Unified Magnetic Equivalent Circuit to model the winding connecting to the core. The simplified magnetic equivalent circuit is depicted in Figure 6

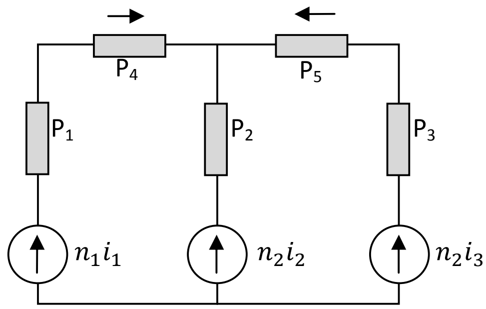


Figure 6 Magnetic Equivalent Circuit of a three-leg core type transformer

The 3x3 inductance matrix [L] for the core above can be found using the UMEC inductance equation

$$[L] = [N][M]_{3\times3}[N] \quad \text{Eq. 24}$$

With $[M]_{3x3}$ as the 3x3 sub-matrix of the matrix

$$[M] = [P] - [P][A]\{[A]^T[P][A]\}^{-1}[A]^T[P] \quad \text{Eq. 25}$$

Where:

[N] : is the turns matrix, given by

$$[N] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Eq. 26}$$

With all the core windings having one turn around each of the core legs

[P] : is a diagonal matrix of permeances for each of the lumped magnetic core elements, where each element is calculated using the expression

$$p = \mu \frac{Area}{Length} \quad \text{Eq. 27}$$

Since the actual values of area and length are not known, the permeance matrix is built in terms of the core aspect ratios, and an equivalent permenace is introduced in order to account for the dimensions correction. It should be noted that the length and area of the core legs are assigned unity value

$$[P] = \mu_{eq} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{rayw}{rlyw} & 0 \\ 0 & 0 & 0 & 0 & \frac{rayw}{rlyw} \end{bmatrix} \quad \text{Eq. 28}$$

Where ***rayw*** is the Ratio of yoke to winding-limb area, and ***rlyw*** is the Ratio of yoke to winding-limb length (see Core aspect ratios Figure 8 and Figure 9)

[A] : is the connections matrix, as defined in [4], [5]. The connections matrix for the three-limb core-type transformer in Figure 6 is

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \quad \text{Eq. 29}$$

The inductance μ_{eq} can be factored out of the permeance matrix such that

$$[P] = \mu_{eq}[P'] \quad \text{Eq. 30}$$

Then Eq. 25 can be re-written as

$$[M'] = [P'] - [P'][A]\{[A]^T[P'][A]\}^{-1}[A]^T[P'] \quad \text{Eq. 31}$$

The inductance equation (Eq. 24) can be re-written as

$$[L] = \mu_{eq}[N][M']_{3\times3}[N] \quad \text{Eq. 32}$$

Please note that in order to factor out the equivalent permeability, it is being assumed that all the sections of core are uniformly saturated. This approximation can introduce some error when the transformer goes into deep saturation.

An expression for the equivalent permeability can be found from the ***unsaturated*** magnetizing test data. For such purpose, Eq. 32 is first written in the form

$$[L] = \mu_{eq}\begin{bmatrix} L1 & M1 & M2 \\ M1 & L2 & M1 \\ M2 & M1 & L1 \end{bmatrix} \quad \text{Eq. 33}$$

with

$$[N][M']_{3\times3}[N] = \begin{bmatrix} L1 & M1 & M2 \\ M1 & L2 & M1 \\ M2 & M1 & L1 \end{bmatrix}$$

Where some of the symmetries in the inductance matrix have been already been reflected.

During the open factory test, a positive sequence voltage is applied to one of the windings and the magnetizing current is measured. In terms of equations this test can be written as

$$\frac{V_{rms}}{\omega}\begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} = i \cdot \mu_{eq}\begin{bmatrix} L1 & M1 & M2 \\ M1 & L2 & M1 \\ M2 & M1 & L1 \end{bmatrix}\begin{bmatrix} \vec{I_1} \\ \vec{I_2} \\ \vec{I_1}a \end{bmatrix} \quad \text{Eq. 34}$$

Where *a* is the 120 degrees shift operator, ie,

$$a = -0.5 + \frac{\sqrt{3}}{2}i = 1\angle 120° \quad \text{Eq. 35}$$

Please note that symmetry between the magnetizing currents for the two outside limbs has been assumed. This only applies for negligible core losses, asymmetry between these currents occurs at high core losses. The magnitude of the magnetizing current corresponding to the center limb (phase B) is naturally different to other two phases due to the particularity of the core magnetic flux path seen by the windings in such limb.

Magnetizing test data in transformers is usually given in terms of the RMS values for voltage and current, where, the current corresponds to the average of the three phases. Therefore, assuming that for a given test point, the average magnetizing current Im_{pu} in per unit is known (see Eq. 36),

$$Im_{pu}\frac{T_{MVA}/3}{V_{RMS}} = \frac{2\left|\vec{I_1}\right| + \left|\vec{I_2}\right|}{3} \circ \qquad \text{Eq. 36}$$

where V_{RMS} is the RMS voltage across the winding (V_{L-L} for delta connected and V_{L-G} for Wye connected windings), then the value of the equivalent permeance can be found by solving the system of equations form by Eq. 34 and Eq. 36. The equivalent permeability is then given by

$$\mu_{eq} = V_{RMS}^2\left[\frac{\sqrt{K1} + 2\sqrt{K2}}{Im_{pu}\cdot\omega\cdot T_{MVA}}\right] \circ \qquad \text{Eq. 37}$$

Where

$$K1 = \frac{2\,L1\,M1 - L1\,M2 - M1\,M2 + L1^2 + M1^2 + M2^2}{L1^2L2^2 + L2^2M2^2 + M1^4 - L1\,L2\,M1^2 - L1\,L2^2M2 - L2\,M1^2M2} \circ \qquad \text{Eq. 38}$$

and

$$K2 = \frac{L2\,M1 + L2^2 + M1^2}{L1^2L2^2 + L2^2M2^2 + M1^4 - L1\,L2\,M1^2 - L1\,L2^2M2 - L2\,M1^2M2} \circ \qquad \text{Eq. 39}$$

A similar procedure can be carried out for the 5-limb core-type transformer. In this case the permeance matrix is defined as

$$[P] = \mu_{eq}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{rayw}{rlyw} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{rayw}{rlyw} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{rayw\times rlyo}{rlyw\times rayo} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{rayw\times rlyo}{rlyw\times rayo} \end{bmatrix} \qquad \text{Eq. 40}$$

Where ***rayo*** is the Ratio of core yoke area to the core outer-limb area, and ***rlyo*** is the Ratio of core yoke length to the core outer -limb length (see Core aspect ratios [4], [5])

The 5-limb transformation matrix [A] is

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad \text{Eq. 41}$$

The expression for the equivalent permeability for the 5-limb transformer is also given by Eq. 37, Eq. 38 and Eq. 39. The definition of the elements L1, M1 and M2 would be different accordingly.

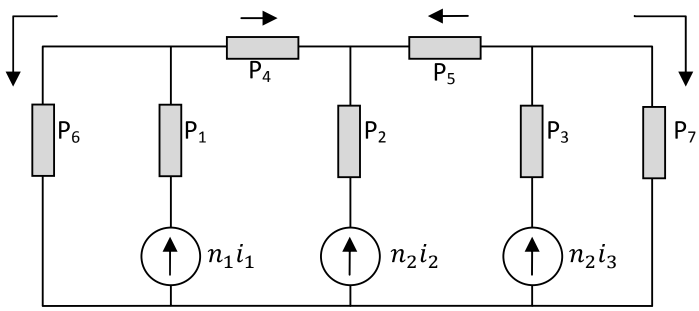


Figure 7 Magnetic Equivalent Circuit of a five-limb core-type transformer. Note that the outer yokes and outer core legs have been merged into single permeances P_6 and P_7

Core aspect ratios

The core aspect ratios for the 3-limb and 5-limb transformers are based on the following dimensions

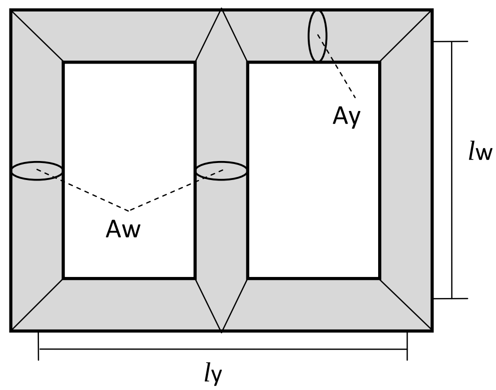


Figure 8 Core dimensions in a three-limb core type transformer

rayw is the Ratio of yoke to winding-limb area, and ***rlyw*** is the Ratio of yoke to winding-limb length

$$rayw = \frac{Ay}{Aw}$$ **Eq. 42**

$$rlyw = \frac{ly}{lw}$$

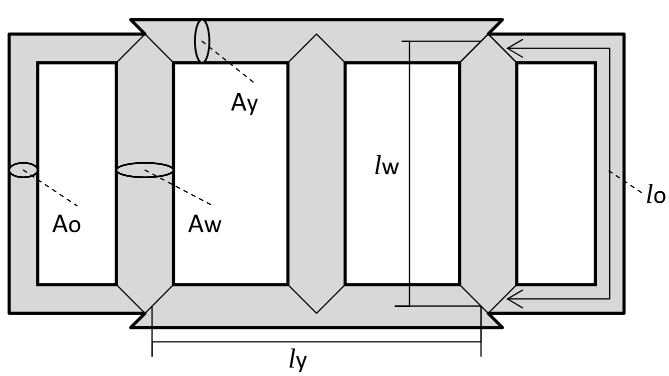


Figure 9 Core dimensions in a five-limb core type transformer

rayo is the Ratio of core yoke area to the core outer-limb area, and ***rlyo*** is the Ratio of core yoke length to the core outer -limb length

$$rayo = \frac{Ay}{Ao}$$ Eq. 43

$$rlyo = \frac{ly}{lo}$$

Recommended default values (when not known) for the core aspect ratios are given in the table below

Core configuration	rayw	rlyw	rayo	rlyo
3-limb	1.0	2.5	N/A	N/A
5-limb	0.65	2.5	1.18	1.0

Table 1 Magnetizing test results with linear core

Leakage impedance between windings and the core winding

A leakage impedance is needed be assigned between the three windings in the transformer and the fictitious core winding. For such purpose, the model first determines which winding is the closest to the core. A leakage impedance *Xc* between the fictitious core winding and the winding closest to the core is assigned as 10% of the smallest per-unit leakage impedance to the winding closest to the core.

$$X_C = 0.1min(X_{12}, X_{13}, X_{23},)°$$ Eq. 44

The impedance between the core winding and the other windings is assigned to be incrementally larger as

$$X_{1c} = X_1 + X_C°$$ Eq. 45

This method is based on the principle that the leakage impedance is proportional to the distance between windings.

The method for determining which winding resides closest to the core is described below

1. The pair of windings with the largest impedance between them is selected. Since the per-unit impedance is proportional to the distance between windings, the pair of windings with the largest leakage impedance between them defines the innermost and outermost windings.
2. Out of the two windings selected above, the one with the lowest voltage is assigned as the coil that resides the closest to the core. In the majority of power transformers designs, the winding with the lowest voltage is placed closest to the core. The reason behind this is that it decreases the probability of an insulation breakdown to the core by reducing and smoothing out the dielectric stresses to the sharp corners of the core steps. It also happens to be the most economical configuration in the majority of cases, since it reduces the need

for large dielectric clearances between the windings and the core, with the consequent reduction in the amount of required copper and core materials.

Provision for other configurations is also provided, where two windings are stacked axially on top of each other, and so on. For more information in this regard please contact support@pscad.com